

Related Rates

Example

1. A circle's area is expanding at a constant rate of $5m^2/s$. How fast is its radius changing when its area is $100\pi m^2$?
2. A spherical meteor is hurtling towards Earth. The angle of how much of the sky it takes up is changing at $1rad/hr$. If we measure the radius of the meteor to be $100m$, how fast is it hurtling towards us when it takes up half of the sky?

Problems

3. A ball of light is falling at a constant rate of $1m/s$. A man who is $2m$ tall is standing $10m$ away. How fast is the length of his shadow changing when the ball is at a height of $4m$?
4. A conical cup that is $6cm$ wide at the top and $5cm$ tall is filled with water is punctured at the bottom and water is coming out at a rate of $10^{-6}m^3/s$. Initially, the cup is filled. How fast is the height of the water changing when the height is $2cm$?
5. A lamppost is $5m$ tall. A woman who is $2m$ tall is walking away from it at a constant rate of $10cm/s$. When she is $2m$ away from the lamppost, how fast is her shadow length changing?
6. Sand is being dumped in a conical pile whose width and height always remain the same. If the sand is being dumped in at a rate of $2m^3/hr$, how fast is the height of the sand changing when the pile is $10cm$ tall?
7. A kite is flying at a current altitude of $100m$. The kite slowly flies further and further away as the string length increases at a rate of $3cm/s$. Assuming the altitude does not change, how fast horizontally is the kite moving when the angle the string forms with the ground is $\pi/6$?
8. A ladder $5m$ tall is lying against a wall. The bottom of the ladder is pulled out at a rate of $10cm/s$. How fast is the area of the triangle formed by the ladder, wall, and floor changing when the bottom of the ladder is $3m$ away from the wall?
9. A conical volcano is $100m$ tall and the base has a radius of $50m$. It is filling with lava at a rate of $\pi m^3/s$. At what rate is the height of the lava rising with it is $50m$ tall?

Optimization

Example

10. Suppose you are trying to make a rectangular fence for your yard. You only have $100m$ of fence but luckily your house borders a straight river, so one side of your rectangular yard will be bordered by a river. What is the largest area yard you can enclose?
11. What is the closest point to $(0, 2)$ on the graph $y = x^2 + 1$.

Problems

12. (4.2, 38) When you cough, the radius of your windpipe decreases and affects the speed of the air through it. If r is the radius of the windpipe, then the speed of the air is $S(r) = ar^2(r_0 - r)$ where a, r_0 are constants. Find the radius r for which the speed is the greatest.
13. You want to construct a cylindrical container that contains $100\pi m^3$ of water. What should the dimensions of the container be if you want to minimize the total surface area?
14. An airline is selling tickets for \$200 each and sells 50 per plane. For every \$10 they decrease the price, they sell 10 more tickets. The plane can hold a maximum of 100 passengers. At what price should they sell their tickets for maximum revenue?
15. Find the rectangle of largest area whose diagonal is of length L .
16. Find the area of the smallest triangle formed by the x axis, y axis, and a line that goes through the point $(4, 2)$.
17. Find the largest rectangle that can be inscribed into a semicircle of radius 1 so that one side of the rectangle is part of the diameter of the semicircle.
18. Suppose you only have $1m$ of wire. You are to construct a circle and a square. What is the maximum and minimum total area of the circle and square?

Tricky Limits

Problems

Solve all of the following questions without using L'Hopital's rule.

19. Find $\lim_{a \rightarrow 2} \frac{a^{2017} - 2^{2017}}{a - 2}$.

20. Find $\lim_{x \rightarrow 1} \frac{e^{3x} - e^3}{x^2 - 1}$.

21. Find $\lim_{x \rightarrow 1} \frac{e^{\sqrt{x}} - e}{x^2 - 3x + 2}$.

22. Find $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x^2 + x}$.

23. Find $\lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x} - \sqrt{4 - x}}$.

24. Find $\lim_{x \rightarrow \infty} \sqrt{x^2 - 4x + 1} - (x + 3)$.